Cambridge Assessment International Education

Scheme of Work

Cambridge IGCSE®
Additional Mathematics 0606
Cambridge O Level
Additional Mathematics 4037

For examination from 2020

Version 1

Cambridge Pathway

In order to help us develop the highest quality resources, we are undertaking a continuous programme of review; not only to measure the success of our resources but also to highlight areas for improvement and to identify new development needs.

We invite you to complete our survey by visiting the website below. Your comments on the quality and relevance of our resources are very important to us.

www.surveymonkey.co.uk/r/GL6ZNJB

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www.cambridgeinternational.org/cambridge-for/teachers/teacherconsultants/

Contents

Introduction

This scheme of work has been designed to support you in your teaching and lesson planning. Making full use of this scheme of work will help you to improve both your teaching and your learners' potential. It is important to have a scheme of work in place in order for you to guarantee that the syllabus is covered fully. You can choose what approach to take and you know the nature of your institution and the levels of ability of your learners. What follows is just one possible approach – you should always check the syllabus for the content of your course.

Suggestions for independent study **(I)** and formative assessment **(F)** are also included. Opportunities for differentiation are indicated as **Extension activities**; there is the potential for differentiation by resource, grouping, expected level of outcome, and degree of support by teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgement of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

Guided learning hours

Guided learning hours give an indication of the amount of contact time you need to have with your learners to deliver a course. Our syllabuses are designed around 130 hours for Cambridge IGCSE and O Level courses. The number of hours may vary depending on local practice and your learners' previous experience of the subject. The table on the following page gives some guidance about how many hours we recommend you spend on each topic area.

Topic	Suggested teaching order	Suggested teaching time (% of the course)
1 Functions	12	≈ 8
2 Quadratic functions	2	≈ 6
3 Equations, inequalities and graphs	5	≈ 3
4 Indices and surds	1	≈ 2
5 Factors of polynomials	6	≈ 4
6 Simultaneous equations	7	≈ 4
7 Logarithmic and exponential functions	3	≈ 6
8 Straight line graphs	4	≈ 4
9 Circular measure	10	≈ 6
10 Trigonometry	11	≈ 12
11 Permutations and combinations	8	≈ 4
12 Series	9	≈ 12
13 Vectors in two dimensions	13	≈ 4
14 Differentiation and integration	14	≈ 25

Recommended prior knowledge

Knowledge of the content of the Cambridge IGCSE Mathematics or Cambridge O Level Mathematics syllabus (or equivalent) is assumed.

Resources

The up-to-date resource list for this syllabus, including textbooks endorsed by Cambridge International, is listed at www.cambridgeinternational.org/
Endorsed textbooks have been written to be closely aligned to the syllabus they support, and have been through a detailed quality assurance process. As such, all textbooks endorsed by Cambridge International for this syllabus are the ideal resource to be used alongside this scheme of work as they cover each learning objective. In addition to reading the syllabus, teachers should refer to the updated specimen assessment materials.

School Support Hub

The School Support Hub www.cambridgeinternational.org/support is a secure online resource bank and community forum for Cambridge teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online. This scheme of work is available as PDF and an editable version in Microsoft Word format; both are available on the School Support Hub at www.cambridgeinternational.org/support. If you are unable to use Microsoft Word you can download Open Office free of charge from www.openoffice.org

Websites

This scheme of work includes website links providing direct access to internet resources. Cambridge Assessment International Education is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services).

The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only the particular resources are recommended.

Command words

The syllabus now includes a list of command words used in the assessment. You should introduce your learners to these words as soon as possible and use them throughout your teaching to ensure that learners are familiar with them and their meaning. See the syllabus for more detail.

How to get the most out of this scheme of work – integrating syllabus content, skills and teaching strategies

We have written this scheme of work for the Cambridge IGCSE Additional Mathematics 0606 / Cambridge O Level Additional Mathematics 4037 syllabus and it provides some ideas and suggestions of how to cover the content of the syllabus. We have designed the following features to help guide you through your course.

Learning objectives from the **syllabus content** help your learners by making it clear the knowledge they are trying to build. Pass these on to your learners by expressing them as 'We are learning to / about…'.

Suggested teaching activities give you lots of ideas about how you can present learners with new information without teacher talk or videos. Try more active methods which get your learners motivated and practising new skills.

…ies

Solve cubic inequalities in the form $k(x-a)(x-b)(x-c) \leqslant d$ graphically

Learners might be given or might need to draw a cubic graph and a line in order to solve a cubic inequality. For example, $y = k(x-a)(x-b)(x-c)$ and the line $y = d$. They need to consider sections of the cubic that are below (or above) the line, and then write the information correctly as inequalities.

Graphing software is essential together with a teacher-led investigative approach. Learners can share ideas and work out what is needed, to deepen their understanding. **(F)**

The 'inequalities' ideas at: https://wiki.geogebra.org/en/Inequalities may be useful if you are making your own resources. Or as a starting point, try the 'Solving polynomial inequalities' resource at www.geogebra.org/m/p7NEtPFy. **(I)**

Learners need to know how to manipulate the inequality they have in order to be able to use it with the graph they are given, for example, using a given graph of the function $y = 2(x-1)(x+3)(x+4)$ to solve $(x-1)(x+3)(x+4) \leqslant 1$. Give learners as much practice as possible to reinforce and extend their skills.

Extension activity:

- Combine the factorisation of a cubic expression with the solution of this typ… given to differentiate between ability levels.
- Give learners a pair of graphs, one cubic and one linear and ask them to fin… using them.

Lots of practice is essential.

Extension activities provide your more able learners with further challenges beyond the basic content of the course. Innovation and independent learning are the basis of these activities.

Independent study (I) gives your learners the opportunity to develop their own ideas and understanding with direct input from you.

Formative assessment (F) is ongoing assessment which informs you about the progress of your learners. Don't forget to leave time to review what your learners have learnt, you could try question and answer, tests, quizzes, 'mind maps', or 'concept maps'. These kinds of activities can be found in the scheme of work.

Past/specimen papers and mark schemes are available to down…

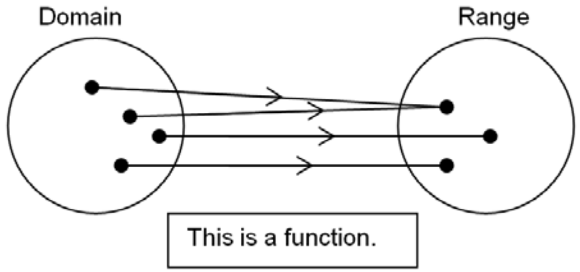
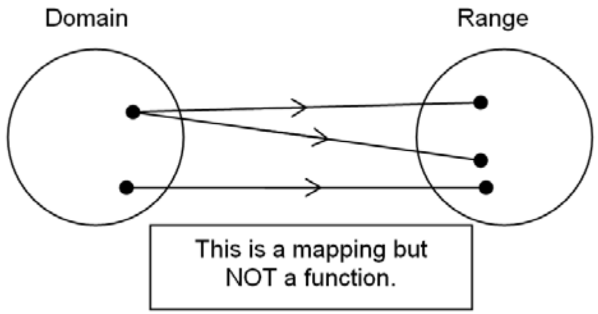
2020 Specimen Paper 1 Q5; 2020 Specimen Paper 2 Q2
Nov 2017 Paper 11 Q3b; Nov 2017 Paper 22 Q3; Nov 2017 Pap…
Jun 2017 Paper 11 Q8; Jun 2017 Paper 22 Q1
Mar 2017 Paper 22 Q1
Nov 2016 Paper 11 Q3; Nov 2016 Paper 21 Q1
Jun 2016 Paper 22 Q10 (including 7 Logarithmic and exponential …
Mar 2016 Paper 12 Q4 (including 1 Functions)

Past papers, specimen papers and **mark schemes** are available for you to download at: www.cambridgeinternational.org/support

Using these resources with your learners allows you to check their progress and give them confidence and understanding.

1 Functions

Syllabus content	Suggested teaching activities
Whole unit	We recommend that learners cover **2 Quadratic functions** before this unit so that the manipulative skills required are established and well-practised. It is possible that trigonometric functions will be used together with the material detailed in this topic, so we also recommend that learners have covered **10 Trigonometry**. The graphing elements of **7 Logarithmic and Exponential Functions** could be done with **1 Functions**, so that the concept of an inverse function being the reflection of the function in the line $y = x$ can be more easily established. The section on understanding the relationship between $y = \mathrm{f}(x)$ and $y = \lvert \mathrm{f}(x) \rvert$, could be done before the rest of the syllabus content for **1 Functions** as part of **3 Equations, inequalities and graphs**. This unit takes the basic skills that have been acquired for Cambridge IGCSE or Cambridge O Level Mathematics and builds on them so that learners are able to handle a variety of functions in more challenging settings. The key skills, notation and concepts relating to functions that learners will have met in IGCSE or Cambridge O Level Mathematics will need to be revisited and formalised with more rigour here.
• understand the terms: function, domain, range (image set), one-one function, inverse function and composition of functions	All of these terms might have been met in Cambridge O Level or Cambridge IGCSE Mathematics. Even if this is the case, they will need formalising in the more challenging settings that learners will encounter in this course. Visual presentations will work better and be more memorable and meaningful for learners – as will relating the function to its graph. Bubble diagrams such as that shown below should be good visual introductions to the key difference between a basic mapping and a function, and the idea can be extended to consider domains and ranges more fully and also one-one functions and subsequently inverses.

Syllabus content	Suggested teaching activities
	  ‘What is a function?’ at www.mathsisfun.com/sets/function.html gives a good solid overview of the basics. Emphasise the difference between a mapping and a function. ‘Is it a function’ by user8 at: www.geogebratube.org/m/1871 will help with this. The first five pages of the ‘Functions and graphs’ resource at: www.stem.org.uk/resources/elibrary/resource/30372/functions-and-graphs is a useful reminder to learners about the structure of functions and the idea of domain and range.
• use the notation f(x) = sin x, f : $x \mapsto \lg x$ ($x > 0$), $f^{-1}(x)$ and $f^{2}(x)$ [= f(f(x))]	Recap notation ready for the more challenging settings that learners will encounter in this course. Composition notation might be new for some learners.

Syllabus content	Suggested teaching activities
• understand the relationship between $y = \mathrm{f}(x)$ and $y = \lvert\mathrm{f}(x)\rvert$, where $\mathrm{f}(x)$ may be linear, quadratic, or trigonometric	It is expected that this will be undertaken with a graphical approach. Learners should both be able to draw the graph of $y = \lvert\mathrm{f}(x)\rvert$ and find the equation of such a function given the graph. Lots of work can be done using GeoGebra software (free to download from www.geogebra.org) to consider graphs such as $y = x^2$ and $y = \lvert x^2\rvert$. Discuss any issues that arise. It is easier to start with straight lines here and consider what is useful information to establish the relationship between the distinct sections of $y = \mathrm{f}(x)$ and $y = \lvert\mathrm{f}(x)\rvert$. 'Understanding absolute value' at: www.geogebratube.org/material/show/id/5538 is a simple way to introduce this concept – starting with $y = 2x - 1$ and adjusting the gradient and intercept to consider various linear functions; you can adjust the gradient and intercept yourself, or ask a learner to do this. Encourage learners to carry out the manual process of drawing graphs of such functions using the ' Graphing Absolute-value functions' resource at: www.purplemath.com/modules/graphabs.htm. This resource also moves on to give an example of what the absolute values of a quadratic function would look like, which is the natural next step. This work could be combined with topic **3 Equations, inequalities and graphs**.
• explain in words why a given function is a function or why it does not have an inverse	Learners should, for example, understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. Again, the use of the bubble diagram can help learners understand this better by visualising, as will the consideration of graphs (for example trigonometric graphs). Both situations where learners are considering whether a mapping is a function or a function has an inverse, can and should be considered using bubble diagrams. Learners should use graphs. Give learners plenty of practice. Initially this should be teacher led but learners should then attempt some work independently. (**I**) Graphical calculators or graphing software such as GeoGebra will be really useful tools here, for example 'Is it a function' and 'Relation or Function?'.

Syllabus content	Suggested teaching activities
• find the inverse of a one-one function and form composite functions	**Inverse function** f^{-1} exists if the function is one-one (so one input value gives only one output value). If $y = f(x)$ then $x = f^{-1}(y)$, so to find the inverse function: 1. Rearrange the formula to make x the subject, so that x = ………. (BEWARE, when taking $\sqrt{\ }$ you must have $\pm\sqrt{\ }$; look at the domain of the original function to determine if you should take the positive or negative root – the x value must fit it.) $x = f^{-1}(y)$, so now the inverse function has been found but with y as the input letter 2. The input letter is usually x, so change the input letter to x 3. Domain (f) = Range(f^{-1}) 4. Range(f) = Domain(f^{-1}) The resource 'Finding inverse functions: quadratic (Ex.2)' at www.khanacademy.org/math/algebra/algebra-functions/v/function-inverses-example-2 provides excellent examples for demonstrating how to find inverse functions whilst being careful of the original domain and also demonstrating why a function and its inverse are reflections in the line $y = x$. **Composite function** Forming the composite function gf means putting the function f into the function g, wherever x was in g before. Composite functions like gf only exist if the range of values coming out of f is allowed as input for the function g. The domain of f might need to be restricted for gf to exist: $\text{Domain}(\text{gf}) \subseteq \text{Domain}(\text{f})$ $\text{Range}(\text{gf}) \subseteq \text{Range}(\text{g})$ Learners should verify by example that gf is not usually the same function as fg. **Extension activity:** For an extra challenge, investigate with learners when fg does not equal gf. The 'Risp 18: When does fg equal gf?' resource on the Rich Starting Points for A Level Mathematics website (Risps www.risps.co.uk) is a great way to investigate and reinforce this concept . (If the link breaks, then from the home page, go to 'The List of Risps from 1 to 40' then scroll to find 'Risp 18: When does fg equal gf?')

Syllabus content	Suggested teaching activities
• use sketch graphs to show the relationship between a function and its inverse	The resource 'Function inverse (Ex.3) at www.khanacademy.org/math/algebra/algebra-functions/v/function-inverses-example-3 provides excellent examples for demonstrating finding inverse functions whilst being careful of the original domain and also demonstrating why a function and its inverse are reflections in the line $y = x$. It should be stressed that, graphically, f and f^{-1} are reflections of each other in the line $y = x$ So for example, if the point (2, 4) lies on f, then the point (4, 2) will lie on the inverse function. Lots of practice (with answers) at finding inverse functions, restricting domains (including trigonometric functions) as well as graphing functions and their inverses can be found at: www.stem.org.uk/resources/elibrary/resource/30372/functions-and-graphs . **(I)** 'Inverse of a function' at www.geogebra.org/m/DV64WEtT is an excellent resource for relating the graph of a function to its inverse.

Past and specimen papers

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(I)(F)**

2020 Specimen Paper 2 Q3 (including 14 Differentiation and integration)
Nov 2017 Paper 11 Q6; Nov 2017 Paper 12 Q6; Nov 2017 Paper 23 Q6
Jun 2017 Paper 11 Q4 (including 7 Logarithmic and exponential functions); Jun 2017 Paper 22 Q9 (including 2 Quadratic functions), Q12
Jun 2017 Paper 23 Q2, Q9
Mar 2017 Paper 22 Q11
Nov 2016 Paper 13 Q1; Nov 2016 Paper 23 Q10
Jun 2016 Paper 11 Q6; Jun 2016 Paper 22 Q11
Mar 2016 Paper 12 Q6 (including 14 Differentiation and integration)
Mar 2015 Paper 12 Q8 (including 14 Differentiation and integration)

2 Quadratic functions

Syllabus content	Suggested teaching activities
Whole unit	Solution of quadratic equations will occur frequently in all types of question. Being able to deal with this effectively is of paramount importance and should be covered early in the course. Learners should show a full method of solution so that they do not to rely on their calculators to provide solutions – the 'solving equation' function on a calculator is useful but should be used as a **check only**. Solution of quadratic equations will also extend to quadratic equations in terms of trigonometric, logarithmic and exponential variables. This can be covered as part of **3 Equations, inequalities and graphs** or when studying each of these particular functions. Learners should already be familiar with the quadratic functions syllabus requirements of Cambridge O Level Mathematics or Cambridge IGCSE Mathematics, specifically, learners should be able to solve quadratic equations by factorisation and either by use of the formula or by completing the square. Solution of simple linear inequalities is also necessary. This unit aims to extend that knowledge by introducing the use of the discriminant to determine the different types of real solutions quadratic equations may have. Use of the completing the square method can be extended to provide an alternative way of determining maximum and minimum values of quadratic functions. This can then be related to **1 Functions** when dealing with domains and ranges. Quadratic inequalities can be dealt with early as a standalone topic. Learners will need good revision of factorisation and completing the square before this. The first two learning objectives can be taught together, as there is some dependence between them. It is also possible to link in the third and part of the fourth learning objective at the same time because of a similar dependence.
• find the maximum or minimum value of the quadratic function $f : x \mapsto ax^2 + bx + c$ by any method	Use mini whiteboards to test learners' prior knowledge. **(F)** Use worksheets, such as those found at www.kutasoftware.com, to revise completing the square in order to determine maximum and minimum values. Other practice material should be used, for example, the 'Quadratics' material at www.khanacademy.org/math/algebra/quadratics **(I)**

Syllabus content	Suggested teaching activities
• use the maximum or minimum value of f(x) to sketch the graph or determine the range for a given domain	Use graphical software to demonstrate graphs of quadratic curves, relating them to maximum and minimum values and hence ranges for given domains. Examples of software include: www.autograph-math.com ; www.desmos.com www.geogebra.org ; http://rechneronline.de/function-graphs Use matching games to reinforce key concepts and extend knowledge of quadratic functions. This could involve: • the use of sketches of quadratic functions • different ways of writing these quadratic functions • maximum and minimum points • coordinates of the points of intersection with the coordinate axes. Prepare cards showing sketches of different quadratic functions in advance. Learners match separate cards showing the different categories above, to each of the different sketches. Incorporate formative assessment opportunities, through observation or a question and answer session. **(F)**
• know the conditions for f(x) = 0 to have: (i) two real roots, (ii) two equal roots, (iii) no real roots & the related conditions for a given line to (i) intersect a given curve, (ii) be a tangent to a given curve (iii) not intersect a given curve	Use graphical software to demonstrate graphs of different quadratic curves and their positions relative to the x-axis and the value of the discriminant for these quadratic curves. Examples of software include: www.autograph-math.com ; www.desmos.com ; www.geogebra.org ; http://rechneronline.de/function-graphs Use worksheets on the use of the discriminant to determine different types of solutions, such as those at: www.kutasoftware.com. Extend these to include questions on the intersection of straight lines and curves.
• solve quadratic equations for real roots and find the solution set for quadratic inequalities	Use mini whiteboards to check prior knowledge of linear inequalities. **(F)** Use graphical software to demonstrate graphs of quadratic curves, relating them to critical values from the solutions of the functions equated to zero and hence to inequalities. Examples of software include: www.autograph-math.com ; www.desmos.com ; www.geogebra.org ; http://rechneronline.de/function-graphs Worksheets to reinforce work covered can be found at: www.kutasoftware.com The resource 'Solving Quadratic Inequalities: Examples' at www.purplemath.com/modules/ineqquad3.htm provides good coverage of the topic basics, together with a self-test option that could be used in a classroom context or on an individual basis. **(I)(F)**

Past and specimen papers
Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(I) (F)** 2020 Specimen Paper 1 Q3 Nov 2017 Paper 13 Q3; Nov 2017 Paper 21 Q1, Q11 (including 14 Differentiation and integration) Jun 2017 Paper 11 Q1; Jun 2017 Paper 22 Q6 Mar 2017 Paper 22 Q4 Nov 2016 Paper 13 Q3; Nov 2016 Paper 21 Q9 Jun 2016 Paper 11 Q1; Jun 2016 Paper 21 Q6 (including 1 Functions) Mar 2016 Paper 12 Q1 Mar 2015 Paper 12 Q2; Mar 2015 Paper 12 Q3 (including 1 Functions) Jun 2016 Paper 21 Q1; Jun 2016 Paper 22 Q1

3 Equations, inequalities and graphs

Syllabus content	Suggested teaching activities
Whole unit	We recommend that learners cover this unit before **1 Functions** as the graphing of functions is often required. However, the second section of **1 Functions** could be studied along with this unit if preferred. We also recommend that learners cover this topic before **6 Simultaneous equations** as they might need to solve quadratic equations in some function of x as part of the solution. Learners should cover **8 Straight line graphs** before starting this unit, as familiarity with straight line graphs is expected. As some equations that are reducible to quadratic form may involve the use of logarithmic or exponential functions, learners should also already have studied **7 Logarithmic and exponential functions**. Learners should have some experience of graphing straight lines and cubic functions through their studies in Cambridge O Level Mathematics or Cambridge IGCSE Mathematics. They should be able to find the intercepts of lines and curves with each coordinate axis. The modulus sign should be familiar to learners as they have used it to denote the magnitude of a vector in Cambridge O Level Mathematics or IGCSE Mathematics and this may be a useful starting point. Consideration of the modulus of a function is given both algebraically and graphically in order to solve equations and inequalities. Learners are expected to identify situations where an equation to be solved is quadratic in some function of x. They should also be aware that this process can lead to solutions that are not valid for the original problem and therefore must be discarded. Learners are also expected to sketch graphs of cubic functions in factorised form and use these to solve cubic inequalities. Although sketches are generally required, learners must be careful with the shapes, especially when drawing the graphs of modulus functions which may involve the drawing of cusps, e.g. see http://mathworld.wolfram.com/Cusp.html.
• solve graphically or algebraically equations of the type $\lvert ax+b\rvert = c \quad (c \geqslant 0)$ and $\lvert ax+b\rvert = \lvert cx+d\rvert$	Learners are more likely to understand what they are trying to do if you start with a graphical approach. Ask learners how the coordinates of particular points on the graph of for $y = 2x+1$ change when the equation changes to $y = \lvert 2x+1\rvert$. Tabulating values and drawing both graphs should enable learners to observe that all the original negative y-coordinates become positive for the modulus function. Graphing software is useful here. For example, www.desmos.com/calculator/bjtt1dx396 allows you to start with $y = \lvert ax+b\rvert$ and change the values of a and b. You can add extra functions to extend the ideas to solving $\lvert ax+b\rvert = \lvert cx+d\rvert$. Use this as a demonstration or as the basis for a learner investigation. Groups of learners investigate the number of expected solutions of the equations $y = ax+b$ with $y = c$ and then

Syllabus content	Suggested teaching activities
	$y = ax + b$ with $y = cx + d$ and consider the impact the modulus has on the number of solutions. They present their group results to the class as a whole, as a formative assessment opportunity. **(F)** Learner can then develop this into the appropriate algebra. Learners should understand that, even when using an algebraic approach, a sketch may be useful, once critical values have been found. Similarly, when solving using a graphical approach, some algebra may be needed if exact values are required. The resource 'Solving Simpler Absolute-Value Equations' at www.purplemath.com/modules/solveabs.htm provides some examples of algebraic solutions. Some equations will need rearranging before they are of the form $\lvert ax+b \rvert = c \quad (c \geqslant 0)$. Learners need to have as much practice as possible in manipulating these when they are solving algebraically. Suitable worksheets are at: https://cdn.kutasoftware.com/Worksheets/Alg2/Solving%20Absolute%20Value%20Equations.pdf (If the link breaks from the home page (www.kutasoftware.com/freeia2.html) go to the 'Equations and Inequalities' section, then select 'Absolute value equations'.) **(I)** When solving equations of the type $\lvert ax+b \rvert = \lvert cx+d \rvert$ algebraically, squaring both sides can lead to a much simpler solution as there are no issues with signs and all the possible values are found more easily (rather than duplicating work or omitting solutions). Lots of practice is essential. **(I)** The 'Absolute value equations' video (Introduction to absolute value equations and graphs) at www.khanacademy.org/math/algebra-home/alg-absolute-value/alg-absolute-value-equations/v/absolute-value-equations is useful for introductory work (although the algebra is considered first and this is used to draw the graph).
• solve graphically or algebraically inequalities of the type $\lvert ax+b \rvert > c \quad (c \geqslant 0)$, $\lvert ax+b \rvert \leqslant c \quad (c > 0)$ and $\lvert ax+b \rvert \leqslant \lvert cx+d \rvert$	Explain solutions of this type using graphs and considering when one graph is below or above the other. You can also cover the section of **1 Functions** that relates to graphing the modulus of linear, quadratic and trigonometric functions here. Solving linear simultaneous equations might also be required. The resource 'Absolute-Value Inequalities' at www.purplemath.com/modules/absineq.htm provides useful examples of $\lvert ax+b \rvert > c \quad (c \geqslant 0)$, $\lvert ax+b \rvert \leqslant c \quad (c > 0)$. The resource 'Absolute value inequalities' (Intro to absolute value inequalities) at www.khanacademy.org/math/algebra-home/alg-absolute-value/alg-absolute-value-inequalities/v/absolute-value-inequalities is useful for introductory work. Suitable worksheets can be found at https://cdn.kutasoftware.com/Worksheets/Alg2/Absolute%20Value%20Inequalities.pdf (If the link breaks from the home page (www.kutasoftware.com/freeia2.html) go to the 'Equations and Inequalities' section, then select

Syllabus content	Suggested teaching activities
	'Absolute value inequalities'.) **(I)** Again, graphing software might be useful. For example, www.desmos.com/calculator/bjtt1dx396 starts with $y=\|ax+b\|$ and allows you to change the values of *a* and *b*. You can add extra functions to extend the ideas to solving $\|ax+b\|=\|cx+d\|$. You could combine this with the work on solving equations. For example, examine the graphs used to see which set of *y* values result in one graph being below the other.
• use substitution to form and solve a quadratic equation in order to solve a related equation	Sometimes learners will be give the substitution, other times they will need to generate their own. Learners will need a great deal of practice to be able to identify the function that should be substituted. Some quadratics will need rearranging even before this stage is reached. When learners have mastered the basic ideas, practise examples of this type. Not all solutions of the resulting quadratic may be valid solutions of the original equation and learners should **always** check whether this is the case. 'Paul's online maths notes' at http://tutorial.math.lamar.edu/Classes/Alg/ReducibleToQuadratic.aspx provides some excellent worked examples and notes. The links for 'Practice Problems' and 'Assignment Problems' at the bottom of the page provide questions for learners to try.
• sketch graphs of cubic polynomials and their moduli, when given in factorised form $y=k(x-a)(x-b)(x-c)$	Learners need to build on their knowledge of the shape of cubic curves. Use the resource 'Finding quadratics from their zeroes' at: www.purplemath.com/modules/fromzero.htm as a basis for discussion about the link between the roots (or zeros) and factors of a polynomial. It starts with a quadratic and then extends, on the next page, to a cubic. Use graphing software or graphical calculators to help learners see the connections and examine the general shapes of these curves. Examples of software include: www.autograph-math.com ; www.desmos.com ; www.geogebra.org ; http://rechneronline.de/function-graphs Useful resources can be found at: www.geogebra.org/materials; search for 'Factorised Cubic Function'. The 'Graphing cubic equations' activity at www.geogebra.org/m/eSSFU4TK is particularly useful as it allows two or three factors to be the same. Discuss the shape of the graph in these cases. It also allows learners to examine the effect of scaling on the *y*-intercept. The activity has in-built investigative questions for learners to consider; learners record results and observations for formative assessment opportunities. **(F)** Graphing the modulus of a cubic equation in factorised form can be done together with the section of **1 Functions** that relates to graphing the modulus of linear, quadratic and trigonometric functions. Practice of as many different cases as possible is essential. **(I)**

Syllabus content	Suggested teaching activities
• solve cubic inequalities in the form $k(x-a)(x-b)(x-c) \leqslant d$ graphically	Learners might be given or might need to draw a cubic graph and a line in order to solve a cubic inequality. For example, $y = k(x-a)(x-b)(x-c)$ and the line $y = d$. They need to consider sections of the cubic that are below (or above) the line, and then write the information correctly as inequalities. Graphing software is essential together with a teacher-led investigative approach. Learners can share ideas and work out what is needed, to deepen their understanding. **(F)** The 'inequalities' ideas at: https://wiki.geogebra.org/en/Inequalities may be useful if you are making your own resources. Or as a starting point, try the 'Solving polynomial inequalities' resource at www.geogebra.org/m/p7NEtPFy. **(I)** Learners need to know how to manipulate the inequality they have in order to be able to use it with the graph they are given, for example, using a given graph of the function $y = 2(x-1)(x+3)(x+4)$ to solve $(x-1)(x+3)(x+4) \leqslant 1$. Give learners as much practice as possible to reinforce and extend their skills. **Extension activity:** • Combine the factorisation of a cubic expression with the solution of this type of inequality, varying the amount of structure given to differentiate between ability levels. • Give learners a pair of graphs, one cubic and one linear and ask them to find two different inequalities that could be solved using them. Lots of practice is essential.

Past and specimen papers

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(I)(F)**

2020 Specimen Paper 1 Q5; 2020 Specimen Paper 2 Q2
Nov 2017 Paper 11 Q3b; Nov 2017 Paper 22 Q3; Nov 2017 Paper 23 Q2
Jun 2017 Paper 11 Q8; Jun 2017 Paper 22 Q1
Mar 2017 Paper 22 Q1
Nov 2016 Paper 11 Q3; Nov 2016 Paper 21 Q1
Jun 2016 Paper 22 Q10 (including 7 Logarithmic and exponential functions)
Mar 2016 Paper 12 Q4 (including 1 Functions)

4 Indices and surds

Syllabus content	Suggested teaching activities
Whole unit	We recommend that learners cover this unit before **14 Differentiation and Integration** since converting from roots to powers is a commonly required skill. We also recommend that learners cover this topic before **7 Logarithmic and Exponential Functions**. Covering this unit before **8 Straight line graphs** will prepare learners for transforming relationships to straight line form. Learners will have some experience of manipulating index terms through their studies in Cambridge O Level Mathematics or IGCSE Mathematics. They will also have met surds when dealing with irrational numbers. Ideally they should be well practised in algebraic techniques for binomial terms – such as the difference of two squares – as this knowledge is also mirrored in this topic. Questions may be set in geometric or algebraic contexts. Learners will need to recall how to find the area and perimeter of shapes such as triangles, rectangles, trapezia and circles. Learners will also need to know how to solve simple equations. Knowledge of basic trigonometry may need to be applied to such questions. This unit builds on skills learners should already have covered in Cambridge O Level Mathematics or IGCSE Mathematics. The examples and questions should be more challenging as learners take a step up in their level of mathematics. It is essential that learners understand that they must show sufficient working to demonstrate that they have fully understood the techniques, rather than relying on their calculator for surd work. Learners should be encouraged to use their calculator only as a checking tool in questions assessing surds.
• perform simple operations with indices and with surds, including rationalising the denominator	Learners will need to revise their knowledge of indices and then develop skills in manipulating surds. Use examples that can be shown to be true using the index laws, in order to convince your learners. Revision and practice exercises are important. **(I)** The 'Surds' exercise at: www.mathsisfun.com/surds.html can be used to reinforce the concept of a surd and give learners some quick practice. It could be used as a starter activity with an advanced group or as a teacher-led activity with a group that needs more support. Learners will need to appreciate algebraically as well as numerically that $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ and vice versa and also $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$ and vice versa. Rationalising the denominator is a new skill dictated purely by mathematical convention. Revise the difference of two squares to show why, for example $1+\sqrt{3}$ is the multiplier to use in order to rationalise $1-\sqrt{3}$. Make sure that

Syllabus content	Suggested teaching activities
	learners appreciate that multiplying both the numerator and the denominator by the 'same' number with opposite sign (the square root conjugate) means that the original expression is being multiplied by a strategic form of 1 and therefore identity is maintained. Use an investigative approach, so that learners discover what rationalising the denominator is all about and why we do it, as well as manipulating expressions with surds. Start with questions such as: simplify $\sqrt{18}$, $4\sqrt{18}-3\sqrt{18}$, $2\sqrt{75}$ to lowest terms. Move on to consider how to simplify terms such as $\sqrt{2}(5-\sqrt{8})$ and then to simplification of expressions such as $\left(2+\sqrt{8}\right)^2$. This will ensure that the level of difficulty is built up gradually. Use an example such as: Simplify each of the following and hence show that all three expressions are equal: (i) $\frac{\sqrt{32}}{4}$ (ii) $4\sqrt{2}-\sqrt{18}$ (iii) $\frac{\sqrt{10}}{\sqrt{5}}$. Start rationalising with single surd denominators, e.g. Write $\frac{3}{\sqrt{5}}$ with a rational denominator. Then extend to binomial terms in the numerator e.g. $\frac{3+\sqrt{5}}{\sqrt{5}}$ and then to binomial terms in both the numerator and denominator, e.g. $\frac{3+\sqrt{5}}{1+\sqrt{5}}$. When learners have acquired the basic skills they can move on to more involved expressions such as $\frac{(3+\sqrt{5})^2}{1+\sqrt{5}}$. When learners have mastered the numerical skills, they can move onto the more challenging skill of applying the laws of indices and roots to algebraic expressions and equations, e.g. Simplify $\frac{\sqrt{4x-3}+(4x-3)^{-\frac{3}{2}}}{\sqrt{4x-3}}$ to solve $\frac{\sqrt{4x-3}+(4x-3)^{-\frac{3}{2}}}{\sqrt{4x-3}}=\frac{5}{4}$

Past and specimen papers

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2020 Specimen Paper 1 Q4
Nov 2017 Paper 22 Q1, Q2; Nov 2017 Paper 23 Q3
Jun 2017 Paper 11 Q3 (including 7 Logarithmic and exponential functions); Jun 2017 Paper 13 Q4; Jun 2017 Paper 21 Q2; Jun 2017 Paper 22 Q2;
Jun 2017 Paper 23 Q1b
Mar 2017 Paper 22 Q5
Nov 2016 Paper 11 Q2; Nov 2016 Paper 13 Q2; Nov 2016 Paper 21 Q2; Nov 2016 Paper 23 Q1; Nov 2016 Paper 23 Q5
Jun 2016 Paper 12 Q4 (including 2 Quadratic functions); Jun 2016 Paper 21 Q5
Mar 2016 Paper 12 Q2; Mar 2016 Paper 22 Q6
Mar 2015 Paper 12 Q10

5 Factors of polynomials

Syllabus content	Suggested teaching activities
Whole unit	We recommend that learners study this unit after **2 Quadratic Functions**. It may help to study this topic before **6 Simultaneous equations**. The work relating factors to roots can be linked with graphing a cubic equation in factorised form in **3 Equations, inequalities and graphs**. Learners should already be familiar with factorising quadratic expressions and solving quadratic equations from Cambridge O Level Mathematics or Cambridge IGCSE Mathematics. These skills will be required for this unit. Learners should also have a good grasp of numerical factors and remainders. Previously learned skills are developed and the examples and questions should be more challenging as learners take a step up in their level of mathematics they are experiencing. Learners should show a full method of solution so that they do not to rely on their calculators to provide solutions – the 'solving equation' function on a calculator is useful but should be used as a **check only**. For example, when asked to factorise a cubic function finding roots, learners should generally **not** use the 'solving equation' function on a calculator and work back to find the factors. Even though it is important to know the relationship between roots and factors, and this should be explored, relying on a calculator in this way does not demonstrate a learner's ability to perform the technique asked for.
• know and use the remainder and factor theorems • find factors of polynomials • solve cubic equations	Clearly define a polynomial, using the 'Polynomial equations' examples at: www.mathwarehouse.com/algebra/polynomial/polynomial-equation.php. Remind learners of quadratic factorising first and show how knowing the roots of the expression enable the factors to be found and vice versa. Next extend the concept to cubics. Multiply three linear factors to show that the result is a cubic expression. Show that knowing a root again enables a factor to be determined. (You can link this with graphing a cubic equation in factorised form in **3 Equations, inequalities and graphs**.) Practise finding roots and writing down the factors from those roots. You could do so using graphical calculators or graphing software such as GeoGebra. Extending this beyond cubics leads to a more formal definition of the factor theorem as: **If a polynomial expression, f(x), is such that f(a) = 0, then ($x - a$) is a factor and vice versa.** Consider methods to factorise a given cubic after one linear factor has been either established or given. Cover different methods of finding a quadratic factor: comparing coefficients; algebraic long division; and synthetic division

Syllabus content	Suggested teaching activities
	using roots rather than factors. These methods can be used to solve cubic equations by factorising, once the factors have been found. Allow learners to use whichever method they feel most comfortable with. A useful explanation of the synthetic division method is 'Synthetic division: The process' which can be found at: www.purplemath.com/modules/synthdiv.htm. For the remainder theorem, the following approach is useful: We have seen with the factor theorem that if $(x - a)$ is a factor of $f(x)$, a cubic equation or expression, then $f(x) = (x - a)$(some quadratic) If, however, $(x - a)$ is NOT a factor of $f(x)$, then $f(x) = (x - a)$(some quadratic) + some remainder Calling the remainder R: $f(x) = (x - a)$(some quadratic) + R However, if we now put $x = a$ we have $f(x) = (a - a)$(some quadratic) + R So the first part, $(a - a)$(some quadratic), has become zero and we are just left with $f(a) = R$ and hence the remainder is actually $f(a)$. We can define the remainder theorem as: **If $x - a$ is not a factor of a polynomial expression, f(x), then f(a) = R, where R is the remainder.** Start with basic skills and build practice. For the basic concepts, see Paul's Online notes (http://tutorial.math.lamar.edu) on 'Factoring polynomials' at: http://tutorial.math.lamar.edu/Classes/Alg/Factoring.aspx **Extension activity:** • For extension beyond cubics, an example of factorising a quintic expression can be found at: www.purplemath.com/modules/solvpoly.htm • Some useful examples of factorising quartics for more able learners are on Paul's Online Notes (http://tutorial.math.lamar.edu) in the section 'Factoring Polynomials with Degree Greater than 2' at: http://tutorial.math.lamar.edu/Classes/Alg/Factoring.aspx

Past and specimen papers
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6 Simultaneous equations

Syllabus content	Suggested teaching activities
Whole unit	We recommend that learners study **2 Quadratic equations** before this unit, since solving quadratic equations is likely to be needed here. It would also be useful for learners to cover **3 Equations, inequalities and graphs** and **5 Factors of polynomials** before this topic. In the case of one linear and one non-linear equation, learners can make the geometric link between the number of solutions of the simultaneous equations and the number of times the line and curve intersect. Solving simultaneous equations will be good revision of the quadratic skills which underpin many areas of the syllabus, so we recommend that it is covered fairly early on. This unit should be covered before **12 Series**. Learners will need to have studied the solution of a pair of simultaneous linear equations for Cambridge O Level Mathematics or Cambridge IGCSE Mathematics. Other essential prior knowledge will be basic skills in working with quadratic functions, algebraic manipulation and solving linear/quadratic equations. Learners' previous skills from Cambridge O Level or IGCSE Mathematics can be used to introduce this topic and clarify/exemplify the number of solutions to be found. However, the emphasis here must be on algebraic processes and learners should understand that accurate drawings are generally not acceptable methods of solution at this level. Learners should show a full method of solution so that they do not to rely on their calculators to provide solutions – the 'solving equation' function on a calculator is useful but should be used as a **check only**.
• solve simultaneous equations in two unknowns by elimination or substitution	Learners should already be familiar with the concept of solving two linear equations by elimination or substitution – this section is an extension of those ideas. Start with one linear and one non-linear equation. For example: Find the points of intersection between the line $y = -3x$ and the curve $x^2 + y^2 = 3$. Start by drawing the graphs of each and then relate this to the algebra. At all times emphasise that the algebra is what is needed here. Extend this to cases where the line is a tangent to the curve and where the line and curve do not intersect, again comparing the graphs with the algebra that arises. In doing this, learners will be able to see that it is possible for there to be varying numbers of solutions, depending on the number of intersections of the line and curve under consideration. Emphasise that clear presentation of written work is essential.

Syllabus content	Suggested teaching activities
	It can be easier to work with one variable when considering the linear equation – encourage learners to work with the simpler option if there is one. Also encourage learners to consider the drawbacks of the elimination method for the solution of two linear equations in cases such as: $\begin{matrix} x-y=2 \\ x^2+y^2=9 \end{matrix}$. Learners quickly understand the need for a method that is more universally applicable, such as the method of substitution. The resource ‘Solving simultaneous non-linear equations graphically’ at www.geogebra.org/m/FHQ3rweH provides an opportunity for learners to see what happens as different lines and graphs of quadratic functions are drawn and what this means in terms of the number of simultaneous solutions of their equations. This activity will make the topic more dynamic for learners. Encourage them to solve their equations algebraically and check they have all the required solutions using the graphing software. The ‘Simultaneous equations’ resource at www.tes.com/teaching-resource/simultaneous-equations-6146699 provides a worksheet (‘One linear and one quadratic’) with structured step-by-step headings. You can develop and add to it to help learners focus on the steps required to complete the task. **(I)** Learners need to understand what the number of intersections of their line and curve means geometrically – for example, that a line which intersects a circle in two points must be a chord, or a line which intersects at one point only is a tangent. Use graphing software such as: www.autograph-math.com ; www.desmos.com ; www.geogebra.org ; http://rechneronline.de/function-graphs or graphical calculators for this purpose. When learners are confident, they can progress to two, simple non-linear equations. Again, substitution of one equation into the other is generally the best approach. Solving simultaneous equations of this type may be required in **12 Series** when, for example, trying to find the value of the common ratio of a geometric progression from information about the value of terms and/or sums of terms. **Extension activity:** The ‘Simultaneous Multiplying’ resource at https://nrich.maths.org/5027 provides a short problem-solving activity for more able learners. **(I)**

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7 Logarithmic and exponential functions

Syllabus content	Suggested teaching activities
Whole unit	The graphing elements of this topic could be done with **1 Functions**, so that the concept of an inverse function being the reflection of the function in the line $y = x$ can be established more easily. Learners should have covered **4 Indices and Surds** before this unit. Learners should have previous knowledge of simple rates of change which is likely to include exponential growth and decay. Knowledge of laws of indices is essential. Some experience of graphing exponential functions from Cambridge O Level Mathematics or Cambridge IGCSE Mathematics is highly desirable. Learners' knowledge of exponential functions can be developed and reinforced using graphing software so that they can visualise the exponential function, which will be a new concept. Logarithms can be introduced in terms of powers. Before starting to tackle this topic, it might help for learners to investigate the history of logarithms – many webpages cover this, such as www.sosmath.com/algebra/logs/log1/log1.html. Learners should be confident enough, knowing the definition of a logarithm and the links to indices, to prove the laws of logarithms and use logarithms to solve equations. Logarithmic functions can then be investigated as the inverse functions of the set of exponential functions.
• know simple properties and graphs of the logarithmic and exponential functions including ln x and e^x (series expansions are not required) and graphs of $ke^{nx} + a$ and $k\ln(ax + b)$ where n, k, a and b are integers	Learners should be familiar with exponential functions from Cambridge O Level or IGCSE Mathematics. You now need to introduce them to the exponential function, e^x. Learners should already have knowledge of the graphs of exponential functions and estimating the gradient of a function by drawing a tangent to the function at a point. Using this, introduce e^x as **the** exponential function whose gradient at the point (0, 1) is equal to 1. 'The exponential function' resource at www.geogebra.org/m/DvTf88mb provides an introduction to this idea. Though e^x is not plotted, it can be seen that as a^x becomes closer to e^x, the gradient of the tangent gets closer to 1. Plot graphs of 2^x, e^x and 3^x and establish the numerical value of e using appropriate software such as GeoGebra or Autograph. **Extension activity:** The resource 'An intuitive guide to exponential functions & e' at https://betterexplained.com/articles/an-intuitive-guide-to-exponential-functions-e/ provides an explanation of e as the unit rate of growth and provides a really good insight for more able learners. Note: Emphasising the "*a*" in a^x as the base of the expression will help learners identify the base of the equivalent logarithm correctly.

Syllabus content	Suggested teaching activities
	Logarithms will be a new presentation of an otherwise familiar concept – powers. Take care to give a clear definition at the start, as learners are easily confused. If $y = a^x$, then we define the logarithm to the base a of y to be equal to x. So the logarithm is the **power** to which a is raised to find y. If $a^x = y$ then $\log_a y = x$ $a > 0$ (is x ←; → log to base a of y) Introduce the idea of a common log (base 10) and logs to various numerical bases to start with and give plenty of examples so that learners start to become comfortable with the concept, e.g. Find the value of $\log_{10}1000$ by asking the question 'To what power must you raise 10 to achieve 1000?' Then introduce natural logs as the solution to, e.g. Find x such that $e^x = 0.59$. Introduce the notation $\log_e y = \ln y$ and start to use the calculator to evaluate expressions, leading to the establishment of: $\log_a 1 = 0$ (and so $\ln 1 = 0$) and $\log_a a = 1$ (and so $\ln e = 1$). Progress to exploring the idea of ln x and e^x as inverse functions. You could this for example using the resource 'Exponential and logarithm functions' at: www.nationalstemcentre.org.uk/elibrary/resource/5883/functions-and-graphs The set of dominoes at www.mmlsoft.com/index.php/products/tarsia is a useful tool to link the inverses of exponential and logarithmic functions and will help learners to make a connection between the two. Learners should be familiar with plotting and sketching graphs of exponential functions. Start with the graphs of $y = e^x$ and $y = \ln x$. Knowledge of the shape and intercepts of each of these graphs is essential.

Syllabus content	Suggested teaching activities
	Introduce transformations one at a time to see what effect they have on shape and intercepts. For example, learners sketch, using whiteboards or graphing software, the graphs of: $y = e^{2x}$ and $y = \ln 2x$ and then $y = 3e^{x}$ and $y = 3\ln x$. Graphing software such as: www.autograph-math.com ; www.desmos.com ; www.geogebra.org ; http://rechneronline.de/function-graphs or graphical calculators can be used. Learners continue until all the required possibilities have been covered. Summarise the results for all learners. Different groups of learners could work on different graphs so that the results found by each group are added to the total information collected. **(F)** The 'Parent functions and Transformations' activity at www.desmos.com/calculator/aixbizuh4n allows you to choose particular functions and then set transformations. You could use it either in groups or with the class as a whole.
• know and use the laws of logarithms (including change of base of logarithms)	Using the laws will involve simplifying expressions and solving simple equations involving logarithms. **Log Law 1 The Addition Law** Demonstrate the proof of $\log_a x + \log_a y = \log_a(xy)$ using $x = a^m$, $y = a^n$ and $xy = a^{m+n}$, the laws of exponents and the basic definition of a logarithm. Learners do not need to memorise the proof but showing it will help to develop understanding. **Log Law 2 The Subtraction Law** Demonstrate the proof of $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$ using $x = a^m$, $y = a^n$ and $\frac{x}{y} = a^{m-n}$, the laws of exponents and the basic definition of a logarithm. **Log Law 3 The Exponent Law** Demonstrate the proof of $\log_a(x^n) = n\log_a x$ using $x = a^m$, $x^n = (a^m)^n = a^{mn}$, the laws of exponents and the basic definition of a logarithm. Even though technically not a law of logarithms, give change of base of logs as: **Log Law 4 Change of Base** Let $x = a^m$ then $\log_a x = m$ and since for all real numbers we should be able to write $a = b^c$ so that $\log_b a = c$ we

Syllabus content	Suggested teaching activities
	have $x = (b^c)^m = b^{cm}$ so $\log_b x = cm$ and $\frac{\log_b x}{c} = m$ and the result $\log_a x = \frac{\log_b x}{\log_b a}$ follows. **Extension activity:** • For a more able group of learners, in teams as them to develop Laws 1 to 3 for themselves, given the simple information detailed above. Give each team a different law to develop and present to the group as a whole. • 'Risp 31: Building log equations' at http://www.s253053503.websitehome.co.uk/risps/risp31.html is a tough but rewarding way to revise logs and their properties. (If the link breaks, from the Risp home page (http://www.s253053503.websitehome.co.uk/risps/index.html) select 'The List of Risps from 1 to 40' then 'Risp 31: Building Log Equations.' **(I)**
• solve equations of the form $a^x = b$	Examples of this kind of equation include: solve $5^x = 7$ or solve $2e^{3x} = 12$. The equations can be solved using the basic definition of a log and then using a scientific calculator to solve to any base. If learners' calculators do not have this capability, they could apply Law 4, changing the base. Alternatively, learners could apply Law 3 and bring down the power and then rearrange. Provide more opportunities to practice by solving equations whose roots are the solutions to equations of the form $a^x = b$. **(I)** **Extension activity:** The activities 'Log Attack' at: http://nrich.maths.org/5831 and 'How Many Solutions?' at: http://nrich.maths.org/334 are interesting **challenges**.

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8 Straight line graphs

Syllabus content	Suggested teaching activities
Whole unit	This unit should be studied after **7 Logarithmic and exponential functions** since the laws of logarithms are essential for transforming to straight line form. Also, as it will be necessary to find points of intersection of two lines, revision of solution of two linear simultaneous equations is desirable. Indeed, material from this unit is often well-combined with material from **6 Simultaneous equations** resulting in questions that will give more able learners a challenge. Learners should be proficient in algebraic manipulation for this unit – for example, they should be able to carry out such processes as rearranging a formula and solving a pair of simultaneous linear equations. The majority of the basic straight line skills used in this unit should have been met in Cambridge O Level Mathematics or Cambridge IGCSE Mathematics. Learners will also need to know some properties of polygons such as special triangles, the kite, rhombus, trapezium and parallelogram. Shapes such as these are useful to assess knowledge of parallel and perpendicular lines. Learners may also need to be able to use efficient methods for finding the area of these polygons. Basic skills regarding two points needs to be learnt. Learners need to cover distance between points, mid-point, equation of and perpendicular lines before proceeding to the more demanding work of reducing relationships to linear form. Problem-solving techniques are important in this unit, with questions being more challenging than at lower levels. **It must be emphasised that at this level, solutions to this type of question by accurate or scale drawing are not acceptable and that more rigorous and accurate methods are expected.**
• interpret the equation of a straight line graph in the form $y = mx + c$	'Lots of lines!' at https://undergroundmathematics.org/geometry-of-equations/lots-of-lines is a starter activity that builds on prior learning. Learners should be able to apply their previously acquired skills to solving challenging problems in this syllabus. The problems they are presented with may have a simple context and the parameters (the gradient and intercept) may need interpreting in terms of that context. **Extension activity:** 'The meaning of slope and y-intercept in the context of word problems' at http://www.purplemath.com/modules/slopyint.htm provides some good examples of interpreting the general equation of a straight line in context.

Syllabus content	Suggested teaching activities
	The 'Two-variable linear equations intro' at www.khanacademy.org/math/math1/math1-two-var-eq is a good individual activity that includes the gradient and intercept from $y = mx + c$ **(I)** Given the equations of two non-parallel lines, learners should be able find their point of intersection using previously acquired skills. Give learners lots of practice to revise these skills, for example, use the resource 'Straight lines' at https://undergroundmathematics.org/geometry-of-equations/describing-straight-lines (I)
• transform given relationships, including $y = ax^n$ and $y = Ab^x$, to straight line form and hence determine unknown constants by calculating the gradient or intercept of the transformed graph	Transforming equations to and from straight line form will be new to learners, and will build on their newly acquired skills in manipulating logarithms and exponentials. Recap on the log laws as a starter activity here. The emphasis should be on the mathematical process of transforming the relationship, rather than interpretation of results when a context is given. Learners should practise using the model under consideration to estimate the value of one of the variables. Give learners lots of practice at setting up linear forms from exponential relationships and then drawing or interpreting graphs of the results. GeoGebra, Autograph or graphical calculators could all be used for this purpose. Texas Instruments (activities) at: http://education.ti.com/en/us/home, for example, have a wealth of resources available. When transforming from a straight line form to a non-linear model, learners might benefit from using the equation $Y = mX + c$ as an intermediate step in their working. Using this form of the equation of the line, learners can take a step-by-step approach. They first find the gradient and y-intercept from the given coordinates. When no diagram is given, encourage learners to draw one. Once the vales of m and c have been inserted into $Y = mX + c$, Y and X can be replaced by the appropriate variables and rearranging when necessary. Learners should not use $y = mx + c$ for this purpose as this leads to incomplete solutions in many cases (they think they have finished when they have not).
• solve questions involving mid-point and length of a line	Learners investigate whether a shape is a particular quadrilateral, e.g. a parallelogram, by considering length of side and slope. Another possibility is finding the equation of the line of symmetry of an isosceles triangle, given the coordinates of its vertices. Learners can practise finding the area/perimeter of a polygon using the self-study resource 'Area & perimenter on the coordinate plane' resource at: www.khanacademy.org/math/geometry/hs-geo-analytic-geometry/hs-geo-dist-problems/e/find-area-and-perimeter-on-the-coordinate-plane **(I)** They can revise simple word problems using the activity 'Coordinate plane word problems: polygons' at www.khanacademy.org/math/geometry/hs-geo-analytic-geometry/hs-geo-dist-problems/e/coordinate-plane-word-problems-with-polygons

Syllabus content	Suggested teaching activities
• know and use the condition for two lines to be parallel or perpendicular, including finding the equation of perpendicular bisectors	Learners need to understand the following: gradients of parallel lines are equal gradients of perpendicular lines are negative reciprocals of each other (i.e. the gradients of perpendicular lines have a product of –1) a perpendicular bisector is a line that is perpendicular to a given line segment and passes through the mid-point of that given line segment. Learners should be able to find: the equation of a line parallel to a given line that passes through a given point the equation of a line perpendicular to a given line that passes through a given point the equation of a perpendicular bisector of a line segment. 'Equations of straight lines' at www.tes.com/teaching-resource/equations-of-straight-lines-6148248 is a tarsia puzzle that can be used to reinforce the skills listed above. Tarsia software can be downloaded for free from at: www.mmlsoft.com/index.php/home/free-software Some more advanced problems with much less structure to guide the learner will often require them to determine and use the parallel and perpendicular line relationships as part of the method. They will need to link together different elements of their knowledge and use them as a cohesive whole to present a logical and reasoned argument. Provide learners with plenty of practice material, for example the 'Simultaneous squares' at https://undergroundmathematics.org/geometry-of-equations/simultaneous-squares **(I)** A collection of excellent examples and practice questions and full solutions can be found at: www.mathscentre.co.nz/Topics/Coordinate%20Geometry/ **(I)**

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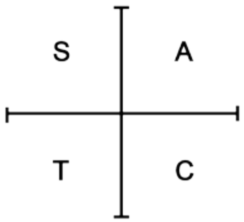
9 Circular measure

Learning objectives	Suggested teaching activities
Whole unit	Learners should study this topic before **10 Trigonometry** where learners are required to solve trigonometric equations in terms of radians. This topic should also be taught before **14 Differentiation and integration**, which can use angles in radians, and where questions might be set that involve using circular measure to form equations in two variables. It is assumed that learners will already be familiar with the following units from the syllabus requirements of Cambridge O Level Mathematics or Cambridge IGCSE Mathematics. <u>Symmetry</u> Learners should be able to use the following symmetry properties of circles: (a) equal chords are equidistant from the centre (b) the perpendicular bisector of a chord passes through the centre (c) tangents from an external point are equal in length <u>Angle</u> Learners should be able to apply the following: angle in a semi-circle; angle between tangent and radius of a circle; angle at the centre of a circle is twice the angle at the circumference; angles in the same segment are equal; angles in opposite segments are supplementary. <u>Mensuration</u> Learners should know how to find the following: the perimeter and area of a rectangle and triangle; the circumference and area of a circle; the area of a parallelogram and a trapezium; arc length and sector area as fractions of the circumference and area of a circle. <u>Trigonometry</u> Learners should be able to: (i) apply Pythagoras Theorem and the sine, cosine and tangent ratios for acute angles to the calculation of a side or of an angle of a right-angled triangle (ii) solve trigonometrical problems in two dimensions (iii) extend sine and cosine functions to angles between 90° and 180° (iv) solve problems using the sine and cosine rules for any triangle and be able to apply the sine rule for the area of a triangle

Learning objectives	Suggested teaching activities
	A knowledge and use of radian measure is essential. Learners should be able to solve problems involving the arc length and sector area of a circle.
• solve problems involving the arc length and sector area of a circle, including knowledge and use of radian measure	Introduction of the radian as an alternate form of measuring angles is an important part of this unit. Encourage learners to think in terms of radians rather than in degrees and then perform a conversion. Matching games where angles in degrees are matched to angles in radians will reinforce the relationship between the two measures. Use Tarsia (www.mmlsoft.com/index.php/home/free-software) for the formation of dominoes or hexagonal jigsaws to test knowledge of degree/radian conversions. Many versions are available on the internet as are worksheets on this topic. A question and answer session at the start of a lesson, asking learners to do conversions and hold up their responses making use of mini whiteboards will provide a formative assessment opportunity. **(F)** Learners should already know how to determine an arc length and sector area using angles in degrees. Learners can use this together with their new knowledge on radians to work out the appropriate formulae in terms of radians for themselves. This could also be done as an investigative piece of individual work. The resource 'Area of sector of a circle' at www.slideshare.net/roszelan/add-mathf4-circular-measure-83 provides the formulae for the arc length and sector area, and a worksheet. **(I)** Give learners problems involving sectors, segments and triangles that require them to extend their knowledge and apply it. Some useful resources 'Radians, degrees and converting between' are available at http://www.purplemath.com/modules/radians.htm and 'Sectors, Areas and Arcs' at http://www.purplemath.com/modules/sectors.htm Problems that cover the entire unit can be found at: www.printableworksheets.rokkada.com/?dq=CircularMeasure. **(I)**

Past and specimen papers
Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(I)(F)** Nov 2017 Paper 11 Q9; Nov 2017 Paper 12 Q10; Nov 2017 Paper 13 Q11 Jun 2017 Paper 11 Q6; Jun 2017 Paper 12 Q10; Jun 2017 Paper 23 Q8 Mar 2017 Paper 12 Q9 Nov 2016 Paper 11 Q8; Nov 2016 Paper 23 Q8 Jun 2016 Paper 21 Q4; Jun 2016 Paper 22 Q6 Mar 2016 Paper 12 Q9 Mar 2015 Paper 22 Q6

10 Trigonometry

Syllabus content	Suggested teaching activities
Whole unit	Learners should cover this unit before **1 Functions** as knowledge of trigonometric functions is very useful for variety in that topic. Solving quadratic equations (generally by factorising) is an essential tool here so learners should already have covered **2 Quadratic Functions**. Also, it is sensible for learners to have studied **9 Circular Measure** before this one, so that the skills in this unit can be practised in both degrees and radians. Learners should have covered all the trigonometric topics from Cambridge O Level Mathematics or Cambridge IGCSE Mathematics. Learners will need algebraic manipulation skills to prove trigonometric identities. They will also need to be able to transform graphs in general as this is a key feature of this unit and the ideas are extended here. This unit builds on the foundation skills learners have already acquired in trigonometry. They start to go beyond the application of the three major trigonometric ratios to acute angles and develop skills needed for general solutions of trigonometric equations, by working with angles of any magnitude. The three minor trigonometric ratios secant, cosecant and cotangent are introduced for the first time. Learners develop graphical skills relating to the transformation and application of trigonometric functions. Simple trigonometric proofs are also introduced.
• know the six trigonometric functions of angles of any magnitude (sine, cosine, tangent, secant, cosecant, cotangent)	'Formation of a Sinusoid from the Unit Circle' at www.geogebra.org/m/ypaPjM8R provides a visual presentation of how the unit circle is linked to the trigonometric functions cosine and sine, and can be used for a teacher-led activity. This is a useful place from which to start building new skills based on the major ratios with which learners are already familiar. Learners need to become familiar with the conventions of an anticlockwise angle being positive and a clockwise angle being negative. When learners have grasped the connections between angles of any magnitude and the periodic nature of the trigonometric functions sine, cosine and tangent, introduce them to the quadrant diagram below.  This diagram will help them recall which trigonometric ratio is positive in which quadrant. Emphasis should be

Syllabus content	Suggested teaching activities
	placed on working from the horizontal axis. Learners could make up their own mnemonic to recall which quadrant is which. The 'CAST Rule' at www.geogebra.org/m/mRdbxDS9 provides a good demonstration. 'Trigonometric ratios of any angle' at www.stem.org.uk/elibrary/resource/34698 is a PowerPoint resource that can help learners to understand circular functions, quadrants, positive and negative angles. The minor ratios sec, cosec (often csc in textbooks) and cot will be new to learners. The relationships with cos, sin and tan are important and various methods of recall exist – one of which is the 3rd letter rule: $\sec A = \dfrac{1}{\cos A}, \quad \text{cosec}\, A = \dfrac{1}{\sin A}, \quad \cot A = \dfrac{1}{\tan A}$ More examples are available in the resource 'Basic trigonometric ratios: examples' at: www.purplemath.com/modules/basirati.htm The video 'Finding reciprocal trig ratios' at www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/example--the-six-trig-ratios is a five-minute presentation where each of the six trigonometric ratios is found from a basic, right-angled triangle. The signs of cosec, sec and cot correspond to those of sin, cos and tan respectively, so make learners aware that, provided that you know the signs of sin, cos and tan, then you should be able to determine the signs of the reciprocal ratios. 'Trace trig functions from the Unit circle' at www.geogebra.org/m/BQ252ctr is a resource that shows how the graphs of all six trigonometric functions are linked to the unit circle, building on what learners have already studied. 'Trigonometric ratios of angles of any size or sign' at http://outreach.mathstat.strath.ac.uk/basicmaths/321_trigratiosforanglesofanysizeorsign.html provides some useful examples. Give learners plenty of practice of finding angles using their calculators if they are given the sec, cosec or cot of the angle (both in degrees and radians). For example www.nationalstemcentre.org.uk/elibrary/resource/5887/trigonometry **(I)**

Syllabus content	Suggested teaching activities
• understand amplitude and periodicity and the relationship between graphs of related trigonometric functions, e.g. sin *x* and sin 2*x*	Amplitude and periodicity relationships between the graphs of transformed trigonometric functions should be an extension of the transformation of graphs generally. Build on this knowledge and, using suitable software, such as GeoGebra or graphical calculators, encourage learners to investigate relationships by graphing related graphs and considering those relationships. This would also provide an opportunity for learners to practise working in radian measure, to increase their familiarity with the concept. This could easily be combined with the next section on drawing transformed graphs.
• draw and use the graphs of: $y = a \sin bx + c$ $y = a \cos bx + c$ $y = a \tan bx + c$ where *a* is a positive integer, *b* is a simple fraction or integer (fractions will have a denominator of 2, 3, 4, 6 or 8 only), and *c* is an integer.	Learners can consider the transformed graphs of sine and cosine functions and explore relationships using the 'Trig graph transformation 3' resource at at: www.geogebra.org/m/HXm8Jv9Z. Learners should now be pulling together all the skills they have been learning in relation to transformed trigonometric graphs. Provide reinforcement and practice to help future recall. You could combine the work in **7 Logarithmic and exponential functions** on transforming exponential and logarithmic graphs. The activity 'Parent functions and transformations' at www.desmos.com/calculator/aixbizuh4n allows you to choose particular functions and then set transformations. Use this or a similar activity either in groups or with the class as a whole. Learners should be able to see the effect of changing the value of *a*. The fractional values of *b* can then be investigated to see how they stretch the graph horizontally. Finally, the effect of integer values of *c* (both positive and negative) can be established.
• use the relationships $\sin^2 A + \cos^2 A = 1$ $\sec^2 A = 1 + \tan^2 A$, $\text{cosec}^2 A = 1 + \cot^2 A$ $\dfrac{\sin A}{\cos A} = \tan A$, $\dfrac{\cos A}{\sin A} = \cot A$ • solve simple trigonometric equations involving the six trigonometric functions and the above relationships (not including general solution of trigonometric equations)	Start this section by simple practice, such as: express $3\cos^2 x - \sin^2 x$ as a single trigonometric ratio. When this skill has been sufficiently established look at simple equations such as: solve the equation $\cos 2\theta = 0.7$ for values of θ between 0 and 2π, giving your answers in radians correct to 3 significant figures. When they have mastered this, learners can move on to solving equations such as: • solve the equation $2\cos^2\theta = 1 - \sin\theta$ for values of θ between 0° and 360°. Ask learners to consider which of the trigonometric ratios they can replace and which they should keep. Once they have recognised the sensible first step, the rest of the method should follow. • solve the equation $\tan\theta = 3\sin\theta$ for values of θ between 0° and 360°.

Syllabus content	Suggested teaching activities
	A common error here is to divide by $\sin\theta$ rather than to factor it out. Learners should give careful attention to this. It may be helpful for learners to graph each side of the equation using appropriate software and consider the points of intersection before considering the algebra of the solution. Learners need to have as much practice as possible in the best approaches and strategies to solving this type of equation. **(I)**
• prove simple trigonometric identities	The identities that learners will be required to prove essentially involve the relationships already covered in this unit. Learners should bear in mind that when proving a relationship such as: $\frac{\sin A}{1+\cos A}+\frac{1+\cos A}{\sin A}\equiv 2\operatorname{cosec}A$ they should work from left to right and not right to left. They should consider what it is that they have to show, e.g. here the two fractions have combined to become a single term, which suggests a common denominator may be a sensible approach. Emphasise to learners the need to produce clear and simple proofs; a double-column approach with statements and reasons is advisable. Provide learners with practice exercises. **(I)**

Past and specimen papers

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(I)(F)**

2020 Specimen Paper 2 Q4, Q11
Nov 2017 Paper 11 Q10; Nov 2017 Paper 12 Q2, Q11; Nov 2017 Paper 13 Q4; Nov 2017 Paper 21 Q2; Nov 2017 Paper 23 Q10
Jun 2017 Paper 13 Q2, Q3, Q7; Jun 2017 Paper 21 Q4, Q11 (including 3 Equations, inequalities and graphs); Jun 2017 Paper 22 Q10
Jun 2017 Paper 23 Q10 (including 3 Equations, inequalities and graphs)
Mar 2017 Paper 12 Q2, Q5; Mar 2017 Paper 22 Q10
Nov 2016 Paper 13 Q8; Nov 2016 Paper 21 Q6
Jun 2016 Paper 11 Q9; Jun 2016 Paper 12 Q5; Jun 2016 Paper 21 Q9; Jun 2016 Paper 22 Q12
Mar 2016 Paper 12 Q11; Mar 2016 Paper 22 Q4, Q9
Mar 2015 Paper 12 Q11; Mar 2015 Paper 22 Q1

11 Permutations and combinations

Syllabus content	Suggested teaching activities
Whole unit	This is a 'standalone' topic. It does not rely on any other of the topics and therefore can be taught at any time during the course. However, learners should preferably cover this topic before **12 Series**, as some reference to combinations will be needed in that unit together with use of appropriate notation when considering binomial expansions. This topic is likely to be completely new for learners. They should be able to think in a logical fashion in order to solve the problems posed. Learners will need to become familiar with, and be able to use, factorial notation. Learners will also need to be able to distinguish between the words 'arrangements', 'permutations' and 'combinations'.
• recognise and distinguish between a permutation case and a combination case	One of the most important aspects of this unit is the ability to distinguish between permutations and combinations. Useful sources of explanation can be found in the resource 'Easy permutations and combinations' at: http://betterexplained.com/articles/easy-permutations-and-combinations/ and 'Counting principle and factorial' at www.khanacademy.org/math/statistics-probability/counting-permutations-and-combinations. A simple introduction to this would be to ask three learners to stand in a line (one combination) and then ask them to see in how many different ways they are able to arrange themselves(six permutations). Extend this to four learners in a line; the rest of the class can help in trying to see a quick and logical way of obtaining all the permutations. Ask the class how many combinations there will be if five learners stood in a line. Keep re-iterating the difference between a permutation and a combination. Use this to introduce the factorial notation and how learners can use their calculators to determine factorials. Repeat the above exercise with a group of four learners and ask the class to see how many ways groups of one, two, three and four can be chosen from the four learners. Keep reinforcing the difference between permutations and combinations. Start to repeat the exercise with a group of five learners and ask the class to see if they can deduce how many ways groups of one, two, three, four and five can be chosen.

Syllabus content	Suggested teaching activities
• know and use the notation *n*! (with 0! = 1), and the expressions for permutations and combinations of *n* items taken *r* at a time	Continue with the activity above and ask learners to see if they can deduce a formula to represent this using factorial notation. Extend this to six and seven objects and then to choosing *r* objects from *n* objects. See if learners can deduce the difference to the formulae obtained if order suddenly becomes important. Make use of online resources/presentations that can be shown to the class, for example 'Easy permutations and combinations' at: http://betterexplained.com/articles/easy-permutations-and-combinations/ and 'Counting principle and factorial' at www.khanacademy.org/math/statistics-probability/counting-permutations-and-combinations.
• answer simple problems on arrangement and selection (cases with repetition of objects, or with objects arranged in a circle, or involving both permutations and combinations, are excluded)	For more complex, challenging problems adopt a practical approach. For example, How many teams of two people can be chosen from five people if two of the five people have to stay together? (combinations). How many ways are there of arranging a certain number of books on a shelf if two books have to be kept together? (permutations) Set worksheets/exercises with lots of straightforward basic examples involving arrangements, permutations and combinations to reinforce the differences. For example, the 'Simple permutations and combinations worksheet' at http://mrnewbatt.wikispaces.com/file/view/MDM4U+U1L4+worksheet.pdf **(I)**

Past and specimen papers

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(I)(F)**

2020 Specimen Paper 2 Q5
Nov 2017 Paper 11 Q8; Nov 2017 Paper 13 Q9; Nov 2017 Paper 22 Q5
Jun 2017 Paper 12 Q8; Jun 2017 Paper 21 Q8; Jun 2017 Paper 23 Q5
Mar 2017 Paper 12 Q6
Nov 2016 Paper 13 Q9; Nov 2016 Paper 21 Q11
Jun 2016 Paper 21 Q10; Jun 2016 Paper 22 Q3
Mar 2016 Paper 12 Q5; Mar 2015 Paper 22 Q2

12 Series

Syllabus content	Suggested teaching activities
Whole unit	Learners should cover **11 Permutations and combinations** before this topic, to the extent that learners are familiar with factorial notation, combinations and the notation ${}^{n}C_{r}$ and/or $\binom{n}{r}$. Finding of first terms, common differences and common ratios might involve solving simultaneous equations, therefore we also recommend that learners have studied **6 Simultaneous equations**. As arithmetic and geometric progressions are new to this syllabus, there is little past paper material with which to practice. These topics have been, and are still included in, the Cambridge International AS & A Level Mathematics 9709 syllabus and the Pure Mathematics 1 papers (Paper 1) should be a good source of material for formative assessment. Learners should already be partially familiar with this topic from the syllabus requirements of Cambridge O Level Mathematics or Cambridge IGCSE Mathematics. Learners should be able to manipulate directed numbers, use brackets and extract common factors, expand products of algebraic expressions and manipulate simple algebraic fractions. Learners should also be able to recognise simple arithmetic and geometric sequences from a list of terms and find simple expressions for *n*th terms of such sequences. Learners extend their knowledge of algebraic manipulation to expanding expressions of the form $(a+b)^{n}$, where n is a positive integer, using the Binomial Theorem. Pascal's Triangle can be used as an initial way of expanding fairly straightforward binomial expansions and these expansions can then be related to the given formula using the work on combinations from **11 Permutations and combinations**. Learners also extend their knowledge of arithmetic and geometric sequences. Work they have covered in Cambridge O Level or Cambridge IGCSE Mathematics for sequences of numbers is translated into algebraic terms. Learners will problem-solve using the structure of each type of sequence. The concept of a series being the sum of the terms of a sequence is introduced and the formulae for these developed. The sum to infinity of appropriate geometric sequence is also considered.

Syllabus content	Suggested teaching activities
• use the Binomial Theorem for expansion of $(a+b)^n$ for positive integer n	If learners are not familiar with Pascal's Triangle, introduce this by expanding $(a+b)^2$, $(a+b)^3$ and $(a+b)^4$ looking at the coefficients of each term when put in an ordered fashion. This will be a good revision of algebraic practice. Ask learners to deduce the expansions of $(a+b)^5$ and $(a+b)^7$. 'The binomial theorem: formulas' is a useful introduction using Pascal's Triangle, which can be found at: www.purplemath.com/modules/binomial.htm . 'The binomial theorem: examples' at: www.purplemath.com/modules/binomial2.htm provides useful examples. Practise some basic expansions by introducing different terms, including negative terms to replace *a* and *b*, making sure that learners understand that if, e.g. $a=3x$, then $a^4=(3x)^4$, **not** $3x^4$, which is a **very** common mistake. Show that a better way is needed for expansions where *n* is a large positive integer, by rewriting the numbers in Pascal's triangle in terms of combinations, using the notation ${}^nC_r$ and/or $\binom{n}{r}$. Show learners where the binomial expansion formula comes from so that they can relate it to the work done in **11 Permutations and combinations**. Many examples can be found on the internet. For example, three presentations that follow on from each other, introducing Pascal's Triangle and linking it to combinations resulting in the binomial formula are at: www.khanacademy.org/math/trigonometry/polynomial_and-rational/binomial-theorem. Provide learners with exercises for practice, for example, 'The binomial theorem' worksheet at www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/The%20Binomial%20Theorem.pdf. **(I)**
• use the general term $\binom{n}{r}a^{n-r}b^r$, $0 \leqslant r \leqslant n$ (knowledge of the greatest term and properties of the coefficients is not required)	Ask learners if they can deduce the general term in the expansion of $(a+b)^n$, looking for patterns which can be applied to other expansions. Extend this to expansions of the type $\left(ax+\frac{b}{x}\right)^n$ and introduce the idea of terms independent of *x*. Worksheets involving plenty of examples are available on the internet and may be used for general practice. **(I) (F)** For example, 'The Binomial theorem' at www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/The%20Binomial%20Theorem.pdf

Syllabus content	Suggested teaching activities
• recognise arithmetic and geometric progressions	Learners will be familiar with an arithmetic progression (AP) as a number sequence with a common difference between terms, and a geometric progression (GP) as a number sequence with a common multiplier (ratio) between terms. At this level, the problems and formulae are based on the **structure** of the sequences rather than the numbers in the sequence. Therefore, both the language and notation used is more formal. As a whole class starter activity, give learners number sequences and ask them, for example, to identify the first term and difference between the terms (the common difference). Also give them the 1st and 5th terms and ask them to find the term-to-term rule. Matching activities linking a number sequence with its first term and common difference could also be used. Create a Tarsia set of dominoes for this purpose or use a simple two-column arrangement on a whiteboard or handout. Both arithmetic and geometric sequences should be explored at this stage. All of this will be revision of prior knowledge. Use the notation '*a*' and '*d*' for the first term and the difference that is common between the terms of an AP. Similarly use, '*a*' and '*r*' for the first term and the multiplier that is common between the terms of a GP. The first four activities from the 'Sequences and series' resource at www.stem.org.uk/resources/elibrary/resource/32369/sequences-and-series will be helpful to use at various stages. This resources uses subscript notation for terms: a is u_1 and so on. Next, start considering the structure of the sequence. At this stage it is important that learners know that, in an AP, the first term is usually referred to as '*a*' and the common difference is usually given as '*d*' as the formulae they will be using are given in terms of these. (Some texts or websites may use subscript notation such as 1st term is u_1, 2nd term is u_2, ..., nth term is u_n and so on.) Learners should understand that they are now going to work with the structure of the sequence as a starting point to solve problems. This helps them understand why the algebra needs to be applied. Once the idea of an AP as a pattern of terms $a,\ \ a+d,\ \ a+2d,\ \ a+3d,\ldots$ has been established, the formula for the nth term can be developed. Similarly, it is important for learners to recognise that, in a GP, the first term is usually referred to as '*a*' and the common ratio (multiplier) is usually given as '*r*'. Once the idea of a GP as a pattern of terms $a,\ \ ar,\ \ ar^2,\ \ ar^3,\ldots$ has been established, the formula for the nth term can be developed.

Syllabus content	Suggested teaching activities
	To help learners deepen their understanding, you could ask them to investigate the development of these patterns of terms in preparation for finding the formulae for the *n*th terms. Free worksheets for learners to practise all the skills they need in these topics – from the basic notation to more complex problem solving – are at: www.kutasoftware.com/freeia2.html . Scroll down to 'Sequences and Series'. (Some questions involve the interpretation of sigma notation which is not required for this syllabus.) **(I)**
• use the formulae for the *n*th term and for the sum of the first *n* terms to solve problems involving arithmetic or geometric progressions	Learners do not need to memorise the formulae as they are given these in the formula sheet at the start of the examination paper. The derivation of each formula is not assessed. However, it is important that learners understand the origins of each formula and know how to use and apply each one. The structure of each *n*th term formulae is relatively simple and most learners should have no difficulty with these. The sum of the first *n* terms formulae are much less intuitive. Develop the formulae for arithmetic progressions first and allow learners the opportunity to practise these before moving on to geometric progressions. 'Arithmetic sequences and sums' at www.mathsisfun.com/algebra/sequences-sums-arithmetic.html provides a simple introduction to AP work (although the sigma notation is not included in this syllabus). The derivation of the formula for the sum to *n* terms is given as a footnote and is useful. There are 10 interactive multiple-choice questions at the bottom of the page for learners to practise their formalised skills. **(I)** The resource 'Sum of n terms of an A.P' at www.geogebra.org/m/nhDAMPgE provides an individual or whole class activity that encourages the correct use of the sum to *n* terms formula: $S_n = \frac{n}{2}(2a + (n-1)d)$ Ask learners why $(n - 1)d$ is used in the AP formulae, instead of $d(n - 1)$. Some learners may not realise that they are in fact the same expression but that $(n - 1)d$ follows the pattern of the terms in the sequence. Learners will need a lot of practice. Set problem-solving activities for pairs or groups of learners. In these, give learners information about, for example, a specific term and sum and ask them to find the first term and common difference. Learners will need to be able to solve simultaneous equations to be able to do this. Give learners two versions of the formula for the sum to *n* terms of an AP. They need to choose which of these formulae is suitable for the information they have. It is sensible to include plenty of practice with both so that learners work equally well with either version. Learners who have a good understanding of the structure should be able to solve problems more easily. **Extension activity:** 'Risp 8: Arithmetic simultaneous equations' at http://www.s253053503.websitehome.co.uk/risps/risp8.html is an interesting activity linking arithmetic sequences and simultaneous equations. **(I)**

Syllabus content	Suggested teaching activities
	When learners have attempted some problems for themselves, summarise the results before moving on to the formulae for a GP. Again, the sum to *n* terms formula will not be intuitive and should be developed even though the derivation of it is not assessed. 'Geometric sequences and sums' at www.mathsisfun.com/algebra/sequences-sums-geometric.html is a simple introduction to GP work (again, the sigma notation is not included in this syllabus). The derivation of the formula for the sum to *n* terms is given as a footnote and is useful. **Extension activity:** The 'Proof sorter – geometric series' activity at https://nrich.maths.org/1398 gives the sum to *n* terms formula as an unordered list that needs to be ordered. This is a challenging and more interactive activity that some learners might enjoy. Again, learners will need a lot of practice. Set problem-solving activities for pairs or groups of learners. In these, give learners information about, for example, a specific term and sum and ask them to find the first term and possible values of the common ratio. Learners will need to be able to solve simultaneous equations to be able to do this. **Extension activity:** Provide learners with examples of terms that are algebraic rather than numerical to add to the challenge. For example: given that the terms $x-2,\quad 2x-4,\quad x+7$ are three consecutive terms of a geometric progression, find the possible values of x and of the common ratio. Other challenging questions can be set combining arithmetic and geometric progressions.
• use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression	Even though the derivation of the sum to infinity of a GP is not assessed, learners should have an understanding of where it comes from. Do this by considering some numerical arithmetic progressions, geometric progressions that diverge and geometric progressions that converge and look at their structure. Once learners have seen practical examples of a GP converging only when $\lvert r \rvert < 1$, the reasoning can be supported by looking at what happens to the sum to *n* terms formula in these cases. Give learners a variety of sequences and ask them to tell you what happens to the sum as the number of terms increases: Does it increase, does it decrease, does it approach a limit? For some learners this will be intuitive. For other learners, graphing the expressions for the sum to *n* terms for some sequences can help them to visualise this. For example, using Geogebra or other graphing software, consider: AP $y=\frac{x}{2}(2(3)+4(x-1))$ for $x > 0$; as x increases then so does y AP $y=\frac{x}{2}(2(3)-5(x-1))$ for $x > 0$; as x increases then y decreases

Syllabus content	Suggested teaching activities
	GP $y=\frac{2(1-3^n)}{1-3}$ for $x > 0$; as x increases then so does y GP $y=\frac{2(1-0.5^n)}{1-0.5}$ for $x > 0$; as x increases then y approaches the value 4. Learners might find it helpful to write the formula for the sum to n terms as the difference of two fractions: $S_n=\frac{a(1-r^n)}{1-r}=\frac{a}{1-r}-\frac{ar^n}{1-r}$. The reason for the condition $\|r\| < 1$ is then much clearer. 'Geometric sequences and sums' at www.mathsisfun.com/algebra/sequences-sums-geometric.html is a summary of the effect on the sum to n terms formula when n tends to infinity. There are also 11 interactive multiple-choice questions at the bottom of the page for learners to practise their formalised skills, including the sum to infinity. **(I)** 'Sum of infinite geometric series' at www.geogebra.org/m/j6QR8z3j is an excellent geometric visual presentation that shows what happens to the sum using sectors of a circle. Learners can see the ever-decreasing size of the 'slice' being added to the 'pie'. Once learners are confident with the basis for the sum to infinity, set problems including the formula for this. Practice at problem solving with all the formulae is essential. Learners will have to think carefully about how to solve each one. Learners who have a good understanding and mastery of the skills required should be able to apply them to these problems. Much practice material is available in textbooks or online. For example, an excellent variety of video and worksheet material that can be used to support learning is at: www.mathcentre.ac.uk/; search for 'Arithmetic and geometric progressions'.**(I)** **Extension activity:** • 'Risp 20: When does $S_n = u_n$?' at http://www.s253053503.websitehome.co.uk/risps/risp20.html is a challenging activity linking sums of terms with values of terms. **(I)** • A variety of support material and some excellent material for extension activities is at. https://undergroundmathematics.org/sequences **(I)**

Past and specimen papers
Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(I)(F)** 2020 Specimen Paper 1 Q9, Q10 Nov 2017 Paper 12 Q3; Nov 2017 Paper 13 Q7; Nov 2017 Paper 21 Q9 (including 2 Quadratic functions) Jun 2017 Paper 12 Q4; Jun 2017 Paper 21 Q5; Jun 2017 Paper 23 Q6 Mar 2017 Paper 12 Q3 Nov 2016 Paper 11 Q4; Nov 2016 Paper 13 Q4 Jun 2016 Paper 12 Q2; Jun 2016 Paper 21 Q8 Mar 2016 Paper 22 Q5; Mar 2015 Paper 12 Q4

13 Vectors in two dimensions

Syllabus content	Suggested teaching activities
Whole unit	Learners should study **13 Vectors in two dimensions** before **14 Differentiation and Integration**, where an application of calculus is made to kinematics. In the first instance, a basic understanding of vectors comes from differentiating between scalar and vector quantities. The movement from an original position to a final position of a translation represents a vector. Vectors have been introduced geometrically in Cambridge O Level Mathematics or Cambridge IGCSE Mathematics and then algebraically. The application of the material to be studied here to many areas of science, engineering and applied mathematics should become apparent through the problem solving activities undertaken. All relevant sections of Cambridge O Level Mathematics or Cambridge IGCSE Mathematics should have been covered, including bearings. Learners will also need trigonometric skills applied to both right-angled and non-right-angled triangles and also be able to apply Pythagoras' Theorem. The vector spaces considered here are two dimensional, not three dimensional. There are two main themes – theoretical applications of vector processes, which may be geometric or algebraic in nature, and simple practical applications involving, for example, velocity vectors.
• use vectors in any form, e.g. $\begin{pmatrix} a \\ b \end{pmatrix}$, $\overrightarrow{AB}$, $\mathbf{p}$, $a\mathbf{i} - b\mathbf{j}$	This needs practising throughout the unit rather than being treated as a separate component. However, it is necessary to be **rigorous** with learner use of symbols for vectors and to make sure they understand the different forms used in text and handwritten mathematics. The Cartesian component form should be the only new presentation.
• know and use position vectors and unit vectors	Learners will already be familiar with position vectors from Cambridge O Level and Cambridge IGCSE Mathematics. They will also already know how to calculate the magnitude of a vector. Here, these skills are developed into consideration of unit vectors. Use Geogebra or similar software to establish geometrically the unit vector in a given direction and then use similarity to establish the following result: For any non-unit vector, **a**, the unit vector in the same direction is given by $\frac{\mathbf{a}}{\lvert\mathbf{a}\rvert}$, where $\lvert\mathbf{a}\rvert$ is the magnitude of **a** found using Pythagoras' Theorem. 'Unit vector' at www.mathsisfun.com/algebra/vector-unit.html provides some simple quiz questions learners can use to practise (the 3D questions should be ignored).

Syllabus content	Suggested teaching activities
	Once you have established the concept of a unit vector, introduce the Cartesian (or rectangular) components with **i** as the unit vector in the positive direction of the *x*-axis and **j** as the unit vector in the positive direction of the *y*-axis. Link these to column vectors and learners should be able to transfer easily between the two forms. 'Post 16: Cartesian components of vectors PPT' at www.tes.co.uk/teaching-resource/Cartesian-components-3004262 is a PowerPoint® introducing Cartesian components and giving some examples for learners to practice
• find the magnitude of a vector; add and subtract vectors and multiply vectors by scalars	Learners should already have experience in finding the magnitude, adding, subtracting and multiplying a single vector by a scalar using column vectors and geometric reasoning problems. 'Maths vectors starter plenary powerpoint' at www.tes.com/teaching-resource/maths-vectors-starter-plenary-powerpoint-6161352 provides some geometric reasoning problems that revise some key concepts. It is intended to be used with a voting or multiple-choice system, but you could easily adapt it into a simple group quiz. **(F)** You can consider these skills using Cartesian components. The type of problem that learners will expect to be able to solve will be more challenging at this level. A basic understanding of vectors comes from differentiating between scalar and vector quantities. The 'Introduction to vectors and scalars' video at www.khanacademy.org/science/physics/one-dimensional-motion/displacement-velocity-time/v/introduction-to-vectors-and-scalars provides an introduction to distance/displacement and speed/velocity. You could use it as the basis of discussion of other types of physical scalar and vector quantities.
• compose and resolve velocities	'Magnitude & direction of a vector' at www.mathwarehouse.com/vectors provides questions on resolving vectors, given direction and magnitude, into Cartesian component form. The final components are given as coordinates. You could develop this into an opportunity to practise writing vectors in *x***i** + *y***j** form. Move on from this to introduce the concept of resolving a velocity vector: vector resolution is taking a vector and breaking it down into two or more component vectors. Vector composition is the process of determining a resultant vector by adding two or more vectors together. The new vector is called the resultant vector. If the quantity is a velocity, then the new vector is a resultant velocity. Resources you can use to build these concepts are at: www.mathsisfun.com/algebra/vectors.html or www.khanacademy.org/science/physics/two-dimensional-motion/two-dimensional-projectile-mot/a/what-are-velocity-components You will also need to relate vectors to bearings.

Past and specimen papers
Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(I)(F)** 2020 Specimen Paper 2 Q9 Nov 2017 Paper 21 Q10 (including 8 Straight line graphs and 14 Differentiation and integration); Nov 2017 Paper 23 Q5 Jun 2017 Paper 11 Q5; Jun 2017 Paper 12 Q3; Jun 2017 Paper 23 Q4 Mar 2017 Paper 12 Q7 Nov 2016 Paper 23 Q9(i)to(iv) Jun 2016 Paper 12 Q3; Jun 2016 Paper 21 Q7 Mar 2016 Paper 22 Q10 Mar 2015 Paper 22 Q5

14 Differentiation and integration

Syllabus content	Suggested teaching activities
Whole unit	An introduction to a very important mathematical topic, learners will realise that the principles involved can be applied to other areas of mathematics. The topic will also provide an insight into mathematics at a higher level. A good knowledge of algebraic manipulative techniques is required. Learners will be required to know that the gradient of a curve at any point is the same as the gradient of the tangent to the curve at that point. We recommend that learners study **2 Quadratic functions**, **4 Indices and surds**, **7 Logarithmic and exponential functions**, **8 Straight line graphs** and **10 Trigonometry** before applying calculus methods to those topics. The topic of calculus is not encountered in Cambridge O Level Mathematics or Cambridge IGCSE Mathematics. An initial introduction to the idea of a derived function is important so that learners can appreciate why they are using certain rules, rather than just applying these rules by rote. As differentiation and integration of trigonometric functions, logarithmic and exponential functions together with general polynomial functions are involved, this unit could be covered in parts with the associated syllabus area, rather than as a 'standalone' unit at the end of the course. First introduce calculus and differentiation using some basic polynomial functions and finding the coordinates of stationary points, equations of tangents and normals to these basic polynomial functions. You could include differentiation of products and quotients in this introduction. Integration can then be introduced as a reverse process of differentiation and related to plane areas. After you have covered these basic principles, complete topics such as exponential functions and trigonometric functions, applying the appropriate differentiation and integration. We recommend leaving the topic of kinematics until the end of the unit as the functions that may be involved cover the entire range in the syllabus. Encourage learners to realise that this work does not involve the equations of linear motion (*suvat* equations) that they may have encountered in physics or science. Other sources of questions on calculus are the Cambridge International Examinations A level Mathematics 9709 Pure Mathematics 1 papers, although you should take care to ensure that the question material is included in the 0606 syllabus.

Syllabus content	Suggested teaching activities
• understand the idea of a derived function	Introduce differentiation by using the function $y = x^2$ and the points P (1, 1) and Q (2, 4). Learners are able to work out the gradient of the chord PQ easily. Ask them to then find the gradient of the chord PQ_1, where Q_1 has coordinates (1.5, 2.25). Ask learners to continue to look at the gradient of chords PQ_2, PQ_3 and so on where the points $Q_2, Q_3, \ldots$ get closer and closer to the point P. Learners should be able to deduce the gradient of the curve at the point P. Extend this to different points on the curve and see if learners can deduce the pattern. Introduce different curves and simple polynomial curves and ask learners to go through the same process. In this way, learners should be able to deduce the required rule for differentiation of x^n, mx^n and related composite functions. Verify this work using graph plotting software such as www.autograph-math.com ; www.desmos.com ; www.geogebra.org ; http://rechneronline.de/function-graphs You could also introduce the idea of differentiation using graph plotting software and repeating the exercise above with the class observing and making deductions or initial suggestions. You could also use graph plotting software to simply plot tangents to different types of curves at different points. Display the equations of these tangents, enabling learners to state the gradient of the tangent and hence make deductions from these results. This could be done as a class demonstration, by groups of learners or by learners individually, depending on the number of computers available. Use mini whiteboards for checking of work and formative assessment. **(F)**
• use the notations $\mathrm{f}'(x), \mathrm{f}''(x), \frac{\mathrm{d}y}{\mathrm{d}x}, \frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left[= \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\right]$	Introduce the relevant notation once learners are aware of the 'rules'. **Extension activity:** For more able learners, work through a formal proof of the above using this notation.
• use the derivatives of the standard functions x^n (for any rational n), $\sin x, \cos x, \tan x$, e^x, ln x, together with constant multiples, sums and composite functions of these	As differentiation of trigonometric functions, logarithmic and exponential functions together with general polynomial functions are involved, this learning objective could be covered in parts with the associated syllabus area, rather than as a 'standalone' topic at the end of the course. Introduce calculus and differentiation using some basic polynomial functions and finding the coordinates of stationary points, equations of tangents and normals to these basic polynomial functions. As good example of this is 'Derivative rules' at www.mathsisfun.com/calculus/derivatives-rules.html 'Differentiation revision' at www.s-cool.co.uk/a-level/maths/differentiation/ is an example of a presentation that you

Syllabus content	Suggested teaching activities
	could also use. **Extension activity:** Formal proofs like those found on 'Differential calculus' www.khanacademy.org/math/calculus could be done to challenge more able learners. Use mini whiteboards for checking of work and formative assessment. **(F)** Provide learners with practice, for example using worksheets such as those found at: www.kutasoftware.com/freeica.html.**(I)**
• differentiate products and quotients of functions	As with the previous learning objective, you can cover this with the appropriate syllabus area: trigonometry, logarithmic and exponential functions, etc. Revisiting differentiation of products and quotients involving these new functions will ensure that learners get plenty of practice. Some useful resources that cover this can be found at: www.s-cool.co.uk/a-level/maths/differentiation/revise-it/the-product-rule-and-the-quotient-rule www.s-cool.co.uk/a-level/maths/differentiation/ www.khanacademy.org/math/calculus Provide learners with practice, for example those found at: www.kutasoftware.com/freeica.html. **(I)** **Extension activity:** Formal proofs may be done to challenge more able learners.
• apply differentiation to gradients, tangents and normals, stationary points, connected rates of change, small increments and approximations and practical maxima and minima problems	As with the previous learning objectives, you can cover this with the appropriate syllabus area. Resources you could use are: www.s-cool.co.uk/a-level/maths/differentiation/ www.khanacademy.org/math/calculus Give learners plenty of practice. There are plenty of examples of all the topics in textbooks, online worksheets and past examination papers. Examples of worksheets can be found at: www.kutasoftware.com/freeica.html
• use the first and second derivative tests to discriminate between maxima and minima	Learners need to be familiar with both tests for maxima and minima (points of inflection are not included). 'First derivative and second derivative of a function' at: www.geogebra.org/m/v4nfnzwk provides a basis for discussion. The first derivative test is likely to be more intuitive as learners should be confident with the sign of the gradient of tangents to the curve either side of the turning point. When using the first derivative test, learners need to clearly define any information stated. Learners often draw tables with various sloping lines without stating clearly in their table headings what each sloping line represents.

<table>
<tr><th>Syllabus content</th><th>Suggested teaching activities</th></tr>
<tr><td></td><td>
For example, if there were a turning point when $x = 1$ the table might look like:

<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td></td></tr>
<tr><td>$\frac{dy}{dx}$</td><td>-ve</td><td>0</td><td>+ve</td><td></td></tr>
<tr><td>gradient</td><td>\</td><td>—</td><td>/</td><td>min at $x = 1$</td></tr>
</table>

Explain the second derivative test using ideas about the gradient decreasing when the turning point is a maximum and the gradient increasing when the turning point is a minimum. Once learners are confident that the second derivative tells you about the rate of change of the **gradient**, whereas the first derivative tells you about the rate of change of the **curve**, they should be equally happy to apply both tests.

When using the second derivative test, it is important to state all the key points and conclude the test. For example, learners sometimes forget to say which value of x they are using in their test, if there is more than one point being tested. Encourage good practice from the beginning.

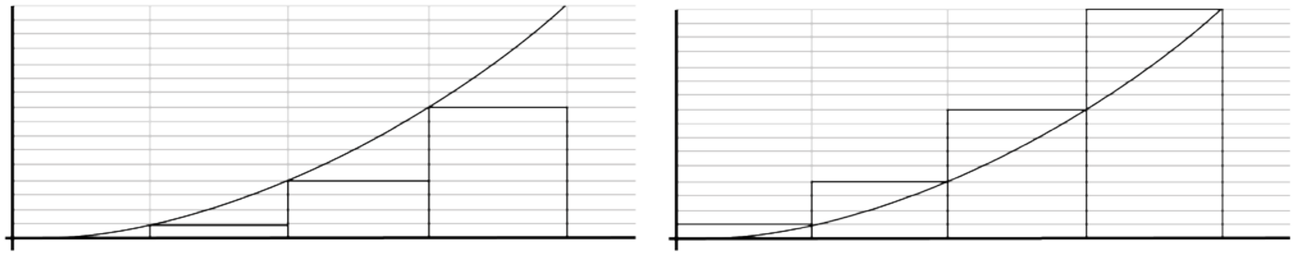
Extension activity: Formal proofs involving the second derivative may be done with more able learners.
</td></tr>
<tr><td>
• understand integration as the reverse process of differentiation
</td><td>
Introduce integration as a reverse process of differentiation using simple questions such as: ‘What would I need to differentiate with respect to x to obtain… ?’

Extend this idea by asking learners to differentiate certain functions of x (for example $x \ln x$) and then deduce an integral (same example, $\int \ln x \, dx$).
</td></tr>
<tr><td>
• integrate sums of terms in powers of x, including $\frac{1}{x}$ and $\frac{1}{ax+b}$
</td><td>
This learning objective should follow straight on from the previous learning objective. The introduction of new notation at this point would be useful.

Emphasise the importance of the constant of integration. ‘Integration revision’ at www.s-cool.co.uk/a-level/maths/integration/ and ‘Differential calculus’ at www.khanacademy.org/math/calculus-home/differential-calculus will be helpful with this.

Use mini whiteboards for checking of work and formative assessment. **(F)**
</td></tr>
</table>

Syllabus content	Suggested teaching activities
	Provide learners with plenty of practice using textbook exercises and worksheets. For example, Worksheets can be found at: www.kutasoftware.com/freeica.html **(I)** Learners should take great care when the power of x is –1. In this case, the rule for inversing differentiation of powers of x fails. However, learners should know what function, when differentiated, gives $\frac{1}{x}$. Encourage them to work out the answer for themselves. Extended to $\frac{1}{ax+b}$ cases. These results are very important and should be practised. This leads into the next learning objective.
• integrate functions of the form $(ax + b)^n$ for any rational n, $\sin(ax + b)$, $\cos(ax + b)$, e^{ax+b}	As integration of trigonometric functions, logarithmic and exponential functions together with general polynomial functions are involved, this learning objective could be covered in parts with the associated syllabus area, rather than as a 'standalone' unit at the end of the course. Introduce calculus and integration using some basic polynomial functions. 'Integration revision' at www.s-cool.co.uk/a-level/maths/integration/ and 'Differential calculus' at www.khanacademy.org/math/calculus-home/differential-calculus will be helpful with this. Use mini whiteboards for checking of work and formative assessment. **(F)** Worksheets for practice are at: www.kutasoftware.com/freeica.html **(I)** 'Integration rules' at www.mathsisfun.com/calculus/integration-rules.html is a useful resource with a table of standard results and some examples as well as some interactive questions for learners to try, including powers of x. **Extension activity:** Formal proofs may be done to challenge more able learners.

Syllabus content	Suggested teaching activities
• evaluate definite integrals and apply integration to the evaluation of plane areas	Use the following method to introduce the relationship between integration and plane areas: Consider an area enclosed by the curve of $y = x^2$, the x-axis and the lines $x = 0$ and $x = 4$. Ask learners to split the area into rectangles as below:  Learners will then be able to deduce that the area A is such that $14 < A < 30$ Extend this exercise by splitting the above areas into greater numbers of rectangles, thus obtaining more accurate estimates. Graph plotting software is able to perform the above operations. You could use it as either a demonstration, with learners individually, or in groups to save time. Suitable graphing software includes www.autograph-math.com ; www.desmos.com ; www.geogebra.org ; http://rechneronline.de/function-graphs A worksheet to introduce the idea of areas being split into rectangles, is at http://kutasoftware.com/freeica.html. **Extension activity:** A formal proof involving the above method could be done involving rectangles of width δx, height y and area δA. For consolidation, give learners past examination questions, textbook questions and online worksheets.
• apply differentiation and integration to kinematics problems that involve displacement, velocity and acceleration of a particle moving in a straight line with variable or constant acceleration, and the use of x–t and v–t graphs	Ask learners for basic definitions of displacement, velocity and acceleration. Ask them to then deduce ways of obtaining velocity and acceleration from a function which represents displacement. Repeat the process for displacement and velocity from a function which represents acceleration. Give learners as much practice as possible, as learners may find some concepts challenging. For example, 'Further maths: calculus in kinematics worksheet' at www.tes.com/teaching-resource/further-maths-calculus-in-kinematics-

Syllabus content	Suggested teaching activities
	worksheet-6147419 (in 2D). 'Model in motion' at www.stem.org.uk/resources/elibrary/resource/31103/model-motion is an activity from the Nuffield Foundation, where learners match descriptions of a variety of real scenarios involving motion with the corresponding velocity–time and displacement–time graphs.

Past and specimen papers

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(I)(F)**

2020 Specimen Paper 1 Q2, Q6, Q7, Q8, Q11
2020 Specimen Paper 2 Q6

Nov 2017 Paper 11 Q5, Q7 (including 7 Logarithmic and exponential functions); Nov 2017 Paper 12 Q4, Q9; Nov 2017 Paper 13 Q2, Q5, Q8;
Nov 2017 Paper 21 Q1, Q5, Q7; Nov 2017 Paper 22 Q6, Q7, Q9, Q11; Nov 2017 Paper 23 Q7, Q9

Jun 2017 Paper 11 Q7, Q9 (including 10 Trigonometry), Q10; Jun 2017 Paper 12 Q2, Q5
Jun 2017 Paper 12 Q6 (including 10 Trigonometry), Q11; Jun 2017 Paper 13 Q5 (including 8 Straight line graphs), Q9, Q10, Q11, Q12
Jun 2017 Paper 21 Q1, Q3, Q12; Jun 2017 Paper 22 Q4, Q5, Q11; Jun 2017 Paper 23 Q7, Q11

Mar 2017 Paper 12 Q8, Q10; Mar 2017 Paper 22 Q9, Q12

Nov 2016 Paper 11 Q5, Q7, Q10; Jun 2016 Paper 12 Q7; Nov 2016 Paper 13 Q6, Q10, Q11; Nov 2016 Paper 21 Q5, Q7, Q8 (including 1 Functions)
Nov 2016 Paper 23 Q3, Q6, Q7

Jun 2016 Paper 11 Q3, Q5,Q11(including 9 Circular measure); Jun 2016 Paper 12 Q6, Q9, Q10, Q11; Jun 2016 Paper 21 Q11, Q12
Jun 2016 Paper 22 Q2, Q9

Mar 2016 Paper 12 Q10; Mar 2016 Paper 22 Q1, Q3, Q11, Q12

Mar 2015 Paper 12 Q6, Q9; Mar 2015 Paper 22 Q4, Q9, Q11

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